

# APPLICATION OF THE HACK INDEX – OR STREAM LENGTH-GRADIENT INDEX (SL INDEX) – TO THE TRACUNHAÉM RIVER WATERSHED, PERNAMBUCO, BRAZIL

Kleython de Araújo MONTEIRO <sup>1</sup>, Ronaldo MISSURA <sup>1</sup>,  
Antonio Carlos de Barros CORREA <sup>2</sup>

(1) Programa de Pós-graduação em Geografia da Universidade Federal de Pernambuco. Avenida Acadêmico Hélio Ramos, S/N 6º andar – Cidade Universitária. CEP 50670-901. Recife, PE.

E-mails: kleython UFPE@hotmail.com; ronaldomissura@hotmail.com

(2) Departamento de Ciências Geográficas da Universidade Federal de Pernambuco. Avenida Acadêmico Hélio Ramos, S/N 6º andar – Cidade Universitária. CEP 50670-901. Recife, PE. E-mail: dbiase@hotmail.com

Introduction  
Characterization of the Area  
Methods and Discussion  
Final Considerations  
Bibliographic References

**ABSTRACT** – For a long time geomorphology has made use of morphological analysis for the study of evolution and interpretation of landforms. In this context, Empirical methods were the major source for the understanding of morphogenetic processes. Notwithstanding, several physical and mathematical methods were envisaged and applied by many authors, mainly after the 1950's decade. Among such methods one was put forward by Hack (1957, 1973) known as *Stream Length-Gradient Index (SL Index)*. In Brazil this method is also referred to as the relation between slope angle and channel extension index (RDE). In this research, this index was applied to the watershed of the Tracunhaem River, one of the coastal drainages of the State of Pernambuco, Northeastern Brazil. The index was applied along the channel of the two most important rivers within the watershed: the Tracunhaem and the Orobó. The data used on the construction of the DTM (Digital Terrain Model) of the basin were obtained from the SB-25-Y-C chart, and the SRTM project, compatible to a 1:250.000 scale. Following, an interpolating method was applied in order to generate the contour lines with 50 m intervals. A drainage raster file was created, compatible with larger numerical scales (1:100.000 e 1:25.000). After, the morphometric index was applied to the channel within reaches limited by the 20 m contour intervals, which generated a spreadsheet with numerical data that later were used to build graphs and charts demonstrating the existence of anomalous zones along the stream channel. Based on the application of this method it was possible to identify the regional knick-point of the basin. Finally the index was correlated to other spatial information, such as geology. This has enabled the interpretation that no major lithological control was particularly connected to the existence of the stream knick-point, which by itself suggests that the retreating escarpment shall be a morphological response to other processes such as faulting or continental margin flexure.

**Keywords:** morphometric indexes, drainage basin, Borborema highlands piedmont, Pernambuco, Brazil.

**RESUMO** – K. de A. Monteiro, R. Missura, A.C. de B. Corrêa - *Aplicação do Índice de Hack – ou Relação Declividade-Extensão (RDE) – na Bacia do Rio Tracunhaém, Pernambuco, Brasil.* Por muito tempo a Geomorfologia tem feito uso da análise morfológica para o estudo da evolução e interpretação do relevo. Neste contexto, os métodos empíricos foram a principal fonte para a compreensão dos processos morfogenéticos. Não obstante, vários métodos físicos e matemáticos foram desenvolvidos e aplicados por inúmeros autores, principalmente após a década de 1950. Entre esses métodos, um foi apresentado por Hack (1957, 1973), conhecido como *Stream-Length Gradient Index (SL Index)*. No Brasil, este método também é conhecido como Relação Declividade-Extensão (RDE). Na presente pesquisa, este índice foi aplicado na bacia do Rio Tracunhaém, uma das bacias costeiras do Estado de Pernambuco, Nordeste do Brasil. O índice foi aplicado ao longo dos canais dos rios mais importantes dentro da bacia hidrográfica: o Tracunhaém e o Orobó. Os dados utilizados na construção do MDT (Modelo Digital de Terreno) da bacia foram obtidos a partir do recorte SB-25-Y-C do projeto SRTM, compatível com a escala de 1:250.000. Em seguida, foi aplicado um método de interpolação a fim de gerar curvas de nível com intervalos de 50 m. Um arquivo *raster* de drenagem foi criado, compatível com escalas maiores (1:100.000 e 1:25.000). Posteriormente, o índice morfométrico foi aplicado ao canal dentro de intervalos altimétricos limitados por curvas de nível de 20 m, gerando uma planilha com dados numéricos que foram usados na construção de gráficos e tabelas que demonstram a existência de zonas anômalas ao longo do canal. Com base na aplicação deste método foi possível identificar o *knickpoint* regional da bacia. Por fim, o índice foi correlacionado com outras informações espaciais, tais como a geologia da bacia. Esta ação permitiu a interpretação de que não há um controle litológico que leve à geração de um *knickpoint* nesta porção da drenagem, o que sugere que a regressão da escarpa deve estar de acordo com a resposta morfológica a outros processos, tais como zonas de falha ou a flexura da borda continental.

**Palavras-chave:** índices morfométricos, bacia de drenagem, Planalto da Borborema, Pernambuco, Brasil.

## INTRODUCTION

The geomorphological sciences, which search the understanding of the landforms, their genesis and evolution, have incorporated the application of several morphometric indexes, in automatized bases, to the analysis of spatial patterns that would allow the inference of structural, lithological and climatic controls on the morphogenesis. Many of these methods are well established in geomorphology, but they have become much more useful and applicable after the introduction of digital processing techniques. Among such methods one should mention the Hack Index, or SL index, which was initially proposed in 1957 (Hack, 1957, 1973).

The morphological analysis was for a long time the most used method for the analysis and interpretation of the landforms. Although it first appeared as a byproduct of the Theory of Geographical Cycle of Davis (1899), it was not fully developed until the mid 20th century, when diverse interpretations in similar deductive bases were proposed. Moreover, the regional set of landforms offered an approximation of how long the relief had been exposed to the atmosphere and how landforms had developed.

However, even before Davis, Gilbert (1877) supported the idea that the landscape analysis should also consider the understanding of three essential factors: materials (rocks and sediments), processes (endogenous or exogenous) and landforms. The interaction of these factors would ultimately generate the physical landscape of the Earth surface.

Starting in the 1960's decade, with a growing application of the systems approach within Geomorphology, the concept of cyclic evolution of the landscape was replaced by more process oriented analysis that focused on the surface dynamics as a key to the understanding of landform development. This

new approach coupled with the growing usage of a mathematical framework led to a large number of efforts towards understanding the landforms by means of the natural processes acting on them, which could be visualized and measured through the use of mathematic language and physical laws.

One of such attempts can be exemplified by Hack (1973) proposal to determine whether a river would be in geomorphological equilibrium based upon the relationship between the river slope and the areal extent of the watershed. This relationship was named by Hack (1973) as the *Stream Length-Gradient Index (SL Index)*. In Brazil, it is also known as the Slope-Extension relationship (Etchebehere et al., 2006).

According to the original proposition, the application of this index would allow the identification of stream sections considered to be "anomalous". The index would be larger or smaller as the stream cut through diverse lithology and structure, displaying different resistances to erosion. In the occurrence of a homogeneous lithological substrate, the anomaly would then be interpreted as related to tectonic movements which would necessarily be younger than the implantation of the drainage network.

This method has been used to validate assumptions of differential erosion, as well as the occurrence of lithological and structural controls. It also became an important tool in the identification of neotectonic deformation along the stream channel network (Etchebehere et al., 2006; Missura, 2005; Monteiro, 2010). According to the above, the present paper tried to identify anomalous zones within the Tracunhaem River Basin, which is the main tributary of the Goiana River, situated in the Northern coastal zone of the state of Pernambuco.

## CHARACTERIZATION OF THE AREA

The watershed of the Tracunhaém River, is a sub-basin of the Goiana River basin. This basin is located in the northeastern portion of the State of Pernambuco, Brazil. The basin has an area of approximately 1300 km<sup>2</sup> and more than 80 km in length from the heads of the Tracunhaém River until its merging with the Capibaribe-Mirim River, when both join forming the Goiana River (Figure 1).

The regional geological-geomorphological framework of this area can be characterized as the passive margin of a flexured platform, structured in Precambrian rocks, more commonly gneisses, migmatites and granites. Many fault structures dated to the Precambrian command the trend of escarpments and

spatial distribution of residual landforms in the area, these fault lines were reactivated in the Cenozoic following the Gondwana breakup (Brito Neves et al., 2001).

Morphologically, this sector of the Northeastern bulge of Brazil, where the study area is located, presents a step-like topography from the coastal plains up to the rocky summits of the Borborema Highlands. These step-like surfaces were previously interpreted as planation surfaces as a consequence of the application of the cyclic landscape forming rationale, specially Lester King's postulates (King, 1952). In the studied basin, two main landscape compartments are found, being neatly differentiated in althimetric sectors separated by a marked knick-point (Figure 2).

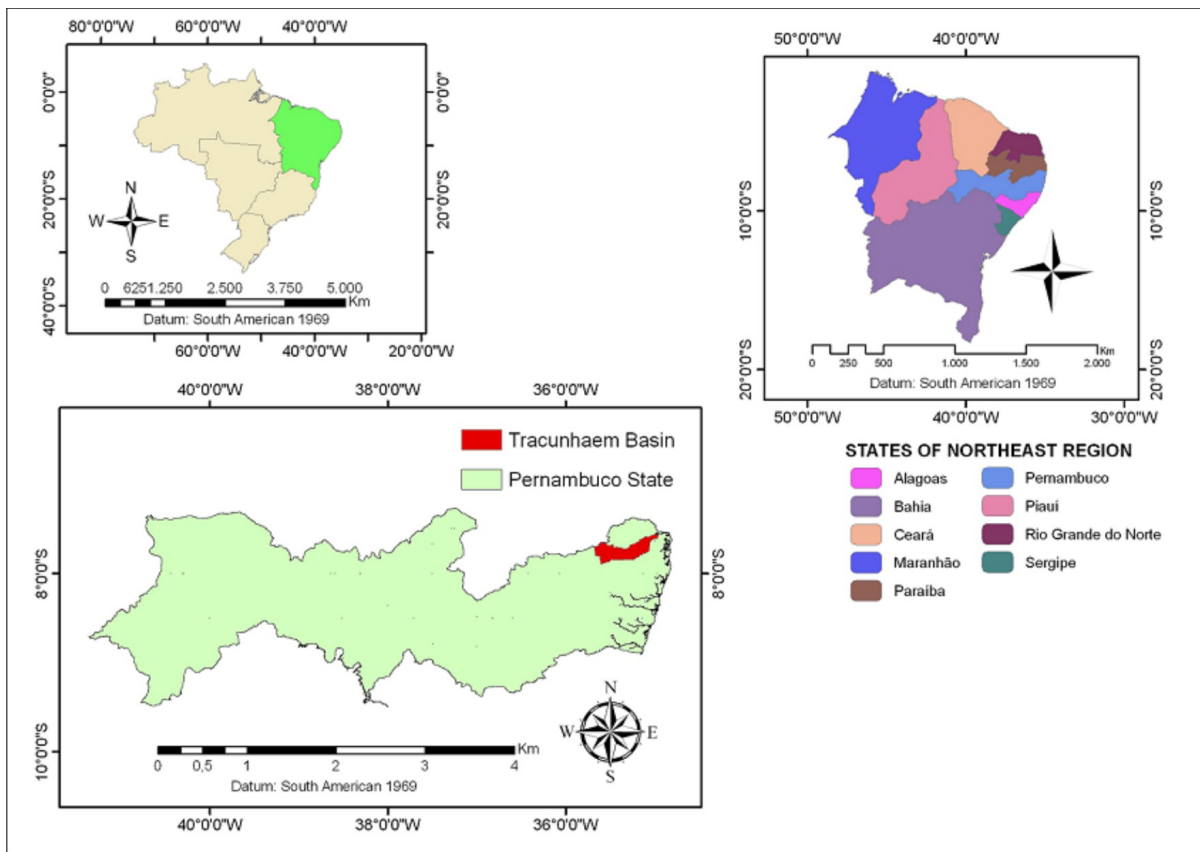


FIGURE 1. The Tracunhaém River Basin.

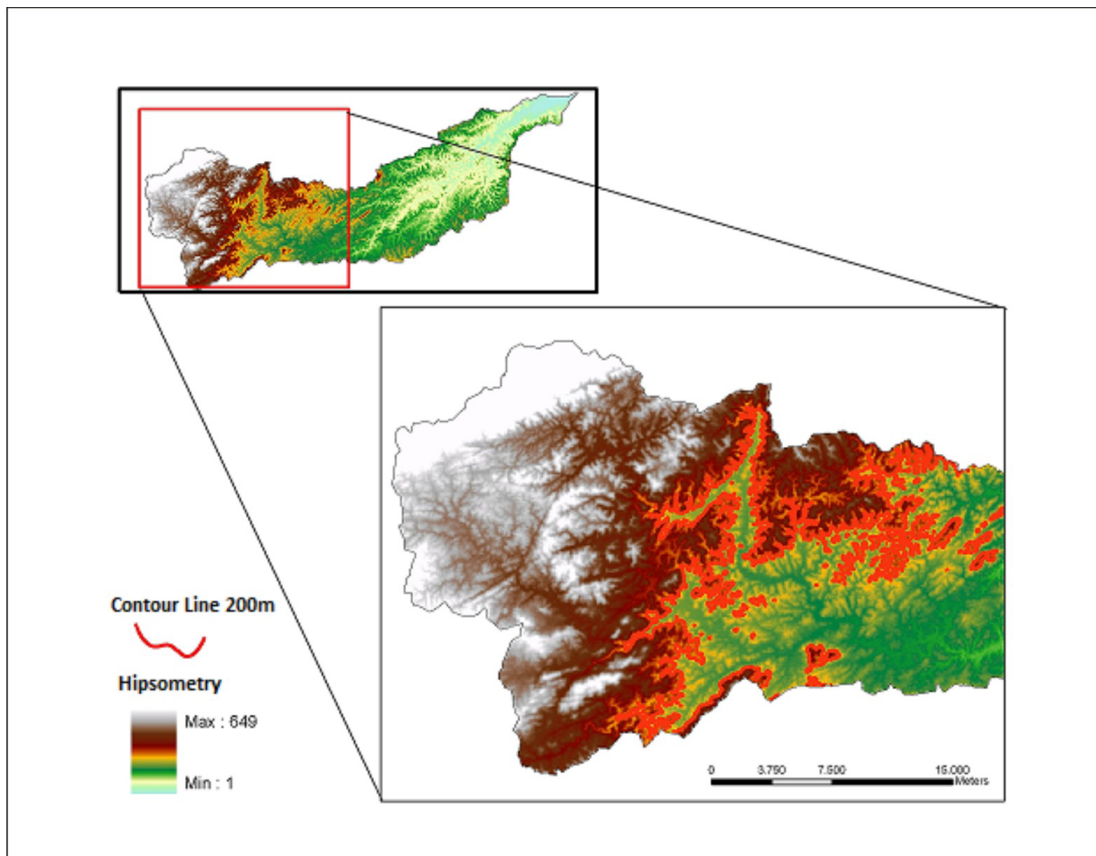


FIGURE 2. The 200 m contour line is neatly separating both compartments.

The first compartment is found in the upper portion of the river, its summit surfaces occur at elevations above 400 m. This sector is also drained by the Orobó River which becomes a tributary of the

Tracunhaem at a lower topographic level. However, the Orobó river runs on a lower topographic compartment, mostly below the 200 m contour line (Figure 2).

## METHODS AND DISCUSSION

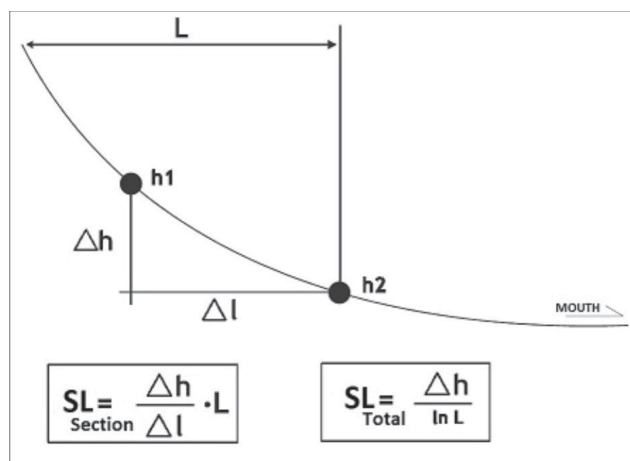
The SL Index was applied to this work, as proposed by Hack (1973) and used by other authors (Troiani and Della Seta, 2007, Etchebehere et al., 2004, 2006; Missura, 2005, Monteiro, 2010), aiming at the identification of morphometric anomalies along the longitudinal profile of the stream and its correlation with controlling factors as lithology and tectonics of the area. The identified anomalies may lead to the understanding of the main morphological discontinuities within the basin, allowing their correlation with lithological as well as structural controls.

The method consists in the application of the following formula:

$$SL = (\Delta h / \Delta l) \cdot L$$

being SL, the Stream-Lenght index,  $\Delta h$  the difference between the highest and the lowest points of a given section of the channel,  $\Delta l$  the horizontal projection of the studied section and L, the length of the farthest point of the studied section to the headwaters of the basin, in a straight line. In order to calculate the SLtotal, one shall divide  $\Delta h$  total of the drainage by the logarithm of L.

This model may be better visualized in the scheme below (Figure 3).



**FIGURE 3.** A scheme showing the calculation of the SL index (from Etchebehere et al., 2006).

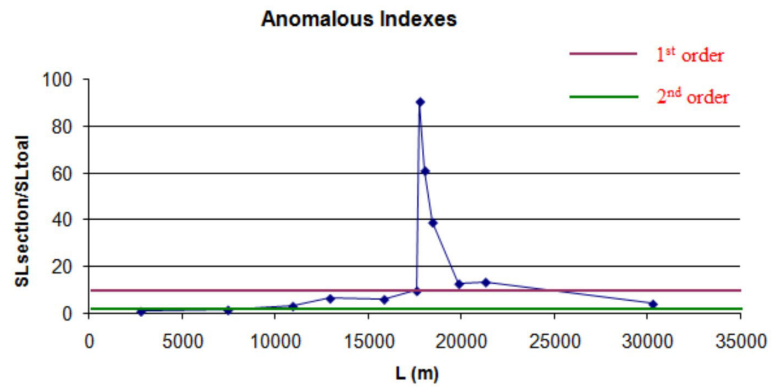
In relation to the anomalous indexes of drainage, Seeber and Gornitz (1983) regard as anomalous, the SLsection divided by the SLtotal. When the result obtained is less than 2 (two) there are no anomalies. When the result is between 2 (two) and 10 (ten) a 2<sup>nd</sup> order anomaly is defined. When the result is equal to or exceeds 10 (ten) a 1<sup>st</sup> order anomaly is defined. This classification implies that where first order anomalies are found, stream channels are very steep, second-order anomalies occur in sectors where stream channels are simply “steep” and when the index is less than two (2) stream channel sectors are “not steep” and the longitudinal profile is said to be graded.

In this paper, the index was applied to the length of the first section of the main channel of the Tracunhaém river and in the Orobó river, its main tributary. Being the longitudinal length of the basin lower than 100 km, it was impossible to apply the index to a larger number of tributaries, since very few of them reached a minimum of 10 km in length which was the lower limit for the application of the technique given the cartographic scale used (1:250.000)

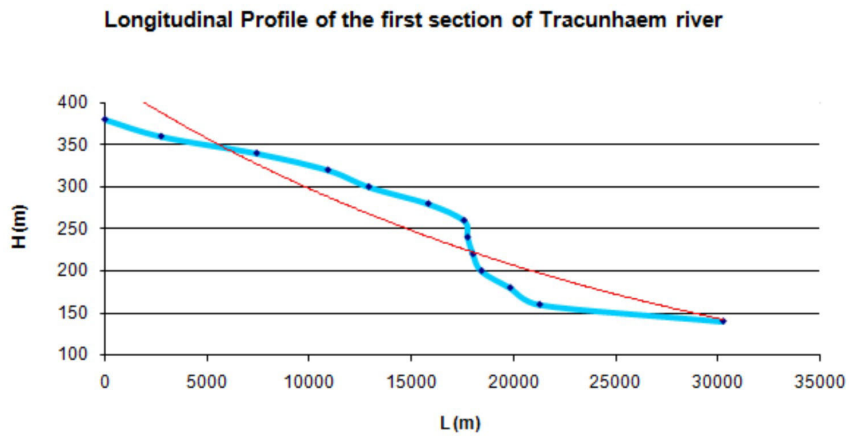
The data used for the construction of the DEM derived from a section of the SB-25-Y-C chart combined with the SRTM Project in a scale of 1:250,000. Later on, an interpolating method was used to generate contour lines at 20 m intervals, considering that this contour interval would be more appropriate for the scale used. A raster file of the drainage network was also generated, compatible with larger scales (1:100,000 e 1:25,000).

Afterwards, the index was applied to the channel in sections were limited by each 20 m contour line, generating a spreadsheet with values, which generated graphs showing the profile of the channel, the values of the SL index, and the identification of the anomalous zones. The graphs and the identified sections with anomalous values are shown in Graphs 1, 2 and 3, respectively.

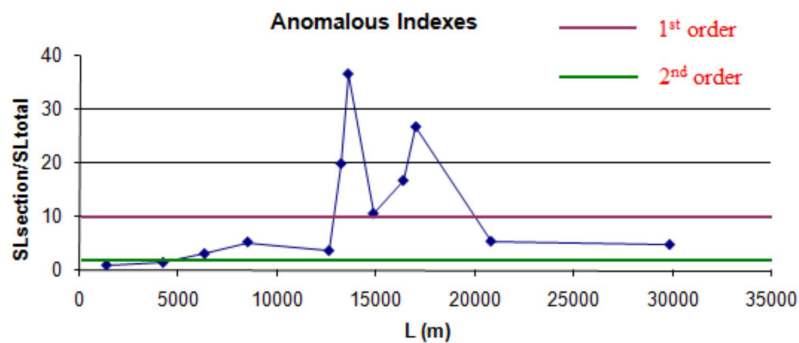
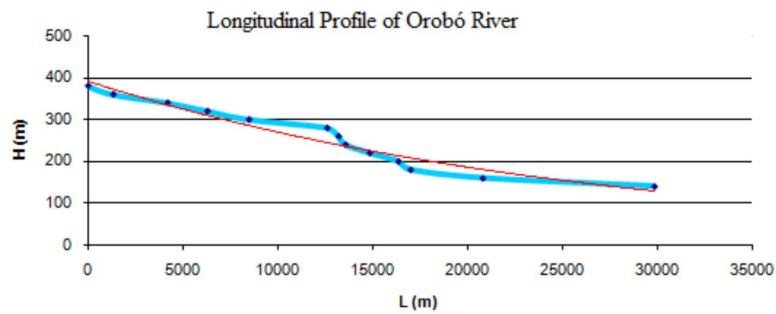
Starting from the application of the proposed methodology, the SL indexes were obtained for the main channel of the Tracunhaém river and for its major tributary, the Orobó river. With the generation of the table of values and the graphs shown below, it was possible to identify anomalous sections along the entire channel. These data showed that there is an area along



**Graph 1.** SL Index of the main channel. Note the anomalous value found in the fourth section of the channel, between 200 and 250 m contour lines.



**Graph 2.** Longitudinal profile of the main channel and the trend line. See that the knickpoint becomes noticeable starting in the fourth section, from where depositional processes become dominant.

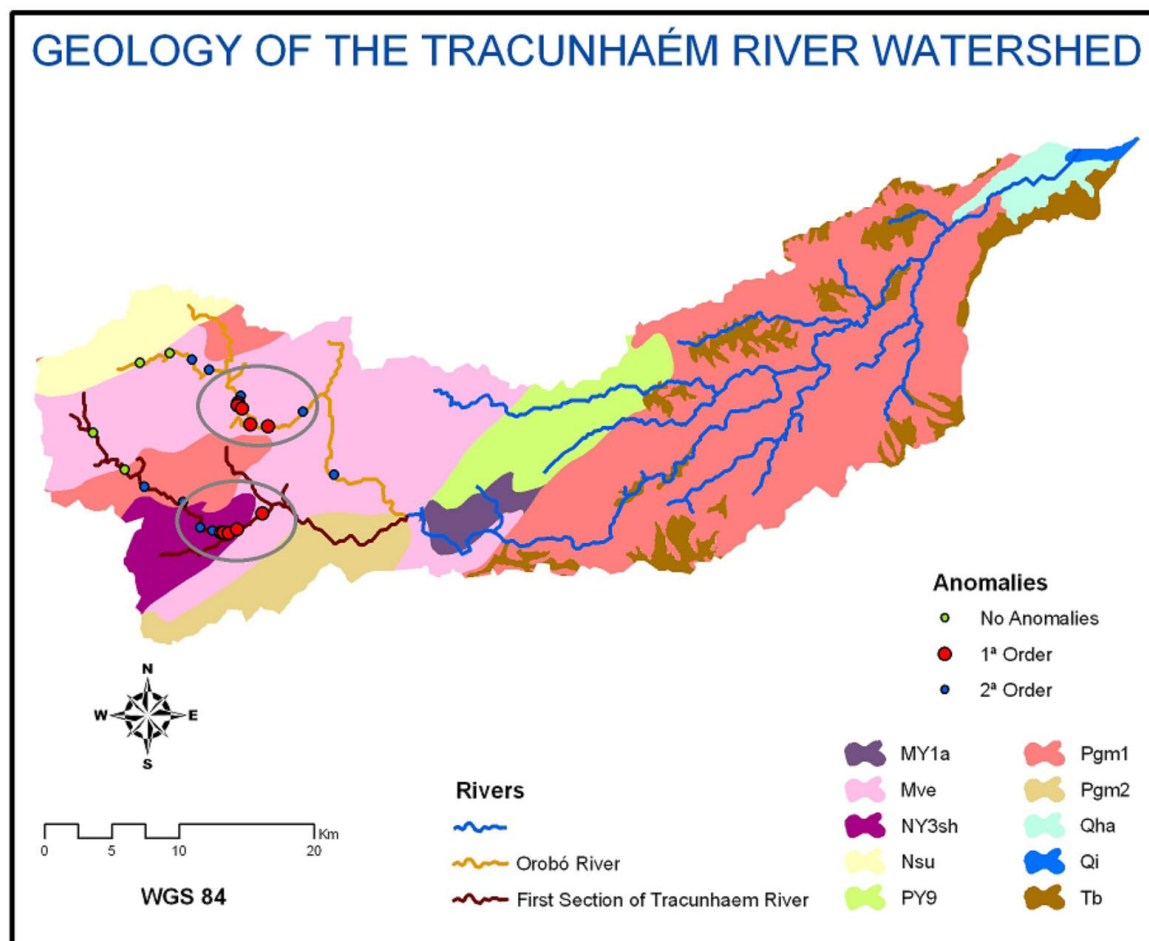


**Graph 3.** SL Index of the Rio Orobo. Note the anomalous values occurring at the same contour line intervals (250 – 200 m a.s.l.).

the channel that can be identified as the regional *knickpoint*, downstream from where depositional processes become dominant (Figure 4).

Another relevant information highlighted by the use of the SL Index was the indication that the occurrence area of anomalies is located along the escarpment line. Overlaying the spatial distribution of the index to the lithological map of the area (Figure 4)

no lithological variations were found which could justify the existence of rock controls on the escarpment (zone of anomaly concentration), that finding suggests that the escarpment could have been generated by faulting or even by continental margin flexure, a process that is supported by many authors, such as Brito Neves et al. (2001) for the sub-littoral zone of the Northeastern bulge of Brazil.



**FIGURE 4.** MY1a –Metaplutonic rocks (Mesoproterozoic): Biotite-Amphibol Augengneisses and Ortogneisses; Mve - Vertentes Complex (Mesoproterozoic): Biotite Schists, Paragneisses; NY3sh – Magmatic suites (Neoproterozoic): Amphibol-Quartz Syenites, Syenites and Quartz Monzonites; Nsu - Surubim Complex (Neoproterozoic): Biotite Gneisses, Granate Biotite Gneisses; PY9 – Intrusive Metagranitoids - Ortogneisses (Paleoproterozoic); Pgm1 – Indiscriminated Gneissic-Migmatitic Complexes (Paleoproterozoic), the more homogenous unit; Pgm2 – Indiscriminated Gneissic-Migmatitic Complexes (Paleoproterozoic), the most migmatized unit; Qha -(Cenozoic - Quaternary): alluvial sediments; Qi - (Cenozoic - Quaternary): sands, silts, clays, of fluvio-lacustrine origin, sands of littoral ridges, Recent and Present, of manglars and sandy eluvial cover; Tb – Barreiras Group (Cenozoic - Tertiary). Note that there is not a variation in rock hardness in the contour line closest to the anomalous zone, which may justify the calculated values, considering that this sector is dominated by metamorphic rocks of similar hardness.

## FINAL CONSIDERATIONS

Morphometric methods, mathematical indexes and statistical analyses became widely used since the rising of geomorphology as an academic science that was interested in finding the existence of geographical

controlling factors by means of application of logical concepts and mathematical modeling. This conceptual concern prompted the emergence of indexes such as those proposed by Hack (1957; 1973), e.g. relief

undercutting, rugosity, and drainage density, among many others. These methods, however, were frequently used with poor criteria and in, certain circumstances, they were presented without any further interpretation, as if they could speak by themselves.

These procedures, however, allow to an enhanced geomorphological analysis when correlations are made, not only with other areas but also with other data in the same area. The greatest difficulties for the application and comparison of the morphometric indexes applied in this work are found when one tries to build correlations with other works, specially because most of the literature deals with mobile belt settings or areas of steep continental escarpments (Troiani and Della Seta, 2007). In mobile belts, the anomalies are related to deformational compressive stress or lithological variations within folded zones, as it occurs with the Appalachians. However, in platform areas, the very few available investigations point to correlations with the geological substratum or with post-orogenic, neotectonic stress.

In the case of the studied area, the identified discontinuities among topographical levels were traditionally interpreted as derived from erosional cycles and not due to the action of tectonics. The differentiation between erosion processes at the escarpment indentations and erosion in the fault scarp becomes difficult with empirical, field observation methods solely, being therefore necessary to use models as the one applied in this paper and the correlation with other data sources, such as geological and/or geophysical, in addition to the recent utilization of Geochronology as an associated parameter in the understanding of the regional morphogenesis. In this regard, works like those of Lima (2008) bring, among other conclusions, that there is no robust relationship between topographic elevation and age of weathering profiles in the Borborema Geological Province. Such findings point to the difficulties in directly associating the step-like landforms with classical models of cyclic retreat of the escarpments.

## BIBLIOGRAPHIC REFERENCES

1. BRITO NEVES, B.B.; SCHMUS, W.R.V.; FETTER, A. Noroeste África-Nordeste do Brasil (Província Borborema). Ensaio comparativo e Problemas de Correlação. **Revista do Instituto de Geociências USP**, v. 1, n. 1, p. 59-78, 2001.
2. DAVIS, W.M. The geographical cycle. **Geographical Journal**, v. 15, n. 5, p. 481-504, 1899.
3. ETCHEBEHERE, M.L.C.; SAAD, A.R.; PERINOTTO, J.A.J.; FULFARO, V.J. Aplicação do Índice “Relação Declividade-Extensão - RDE” na Bacia do Rio do Peixe (SP) para detecção de deformações neotectônicas. **Revista do Instituto de Geociências USP**, Série Científica, São Paulo, v. 4, n. 2, p. 43-56, 2004.
4. ETCHEBEHERE, M. L.C.; SAAD, A.R.; SANTONI, G.C.; CASADO, F.C.; FULFARO, V.J. Detecção de prováveis deformações neotectônicas no vale do Rio do Peixe, região ocidental paulista mediante aplicação de índices RDE (Relação Declividade-Extensão) em segmentos de drenagem. **Geociências**, v. 25, p. 271-289, 2006.
5. GILBERT, G.K. **Report on the geology of the Henry Mountains** (Utah, U.S.A.). U.S. Geographical and Geological Survey of the Rocky Mountain Region (Powell), 160 p., 1877.
6. HACK, J.T. Studies of longitudinal stream profiles in Virginia and Maryland. U.S. Geol. Survey, **Professional Paper 294**, p. 45-97, 1957.
7. HACK, J.T. Stream-profile analysis and stream-gradient index: U.S. Geological Survey. **Journal Research**, v. 1, n. 4, p. 421-429, 1973.
8. KING, L.C. **Morphology of the Earth**. Edinburgh: Oliver, 699 p., 1967.
9. LIMA, M.G. **A História do Intemperismo na Província Borborema Oriental, Nordeste do Brasil: Implicações Paleoclimáticas e Tectônicas**. Natal, 2008, 461 p. Tese (Doutorado) – Universidade Federal do Rio Grande do Norte. Unpublished.
10. MISSURA, R. **Análise morfoestratigráfica da Bacia do Ribeirão dos Poncianos/MG**. Rio Claro, 2005. 136 p. Tese (Mestrado) – Universidade Estadual Paulista. Unpublished.
11. MONTEIRO, K.A. **Superfícies de aplainamento e morfogênese da bacia do rio Tracunhaem, Pernambuco**. Recife, 2010. 124 p. Tese (Mestrado) – Universidade Federal de Pernambuco.
12. SEEBER, L. & GORNITZ, V. River profiles along the Himalayan arc as indicators of active tectonics. **Tectonophysics**, v. 92, p. 335-367, 1983.
13. TROIANI, F. & DELLA SETA, M. The Use of the Stream Length-Gradient Index in morphotectonic analysis of small catchments: A case study from Central Italy. **Geomorphology** (2008), DOI: 10.1016/j.geomorph.2007.06.020, 2007.

*Manuscrito Recebido em: 1 de setembro de 2010  
Revisado e Aceito em: 8 de outubro de 2010*